ECON0106: **Microeconomics**

Problem Set 6

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Question 1. Let X be a finite set of alternatives. Recall two properties of a Stochastic Choice Function (SCF) ρ:
(1) (Stochastic) Monotonicity: For any $x \in B \subseteq A \subseteq X$, $\rho(x, B) \geq \rho(x, A)$.
(2) Weak Stochastic Transitivity (WST): For any $x, y, z \in X$, if $\rho(x, \{x, y\}) \geq 1/2$ and $\rho(y, \{y, z\}) \geq 1/2$, then $\rho(x, \{x, z\}) \geq 1/2$. Fix $X = \{x, y, z\}$. Prove or find a counterexample:

(a) Monotonicity implies Weak Stochastic Transitivity.

(b) Weak Stochastic Transitivity implies Monotonicity.

(c) Comment on what your findings imply for stochastic choice representations.

Question 2. Let $\mathscr{R} := \cup_{N \in \mathbb{N}} \mathbb{R}^N$ and, for $r \in \mathscr{R}$, let $N(r)$ denote the length of the vector, and write $[N(r)] := \{1, \ldots, N(r)\}$. A quantal response is a mapping $Q : \mathscr{R} \to \Delta(\mathbb{N})$ such that $\text{supp}(Q(r)) \subseteq [N(r)]$. We write $Q_n(r)$ to denote the probability that $Q(r)$ assigns to $n \in \mathbb{N}$ and r_n denotes the n-th entry of a vector $r \in \mathbb{R}^N$.

A quantal response function is a reduced-form generalisation of deterministic choice, most commonly employed in game theory. It takes a vector denoting the payoffs of different alternatives and maps it to a distribution over alternatives.

Consider the following restriction:

Definition 1. A quantal response Q is regular iff it satisfies
(1) differentiability: for any $N \in \mathbb{N}$, Q restricted to $\mathbb{R}^N$ is differentiable;
(2) interiority: $Q(r) \in \text{int}\,\Delta([N(r)])$; (3) monotonicity: $\forall r$, $r_n - r_m \geq (>) 0 \iff Q_n(r) - Q_m(r) \geq (>) 0$; and (4) responsiveness: $\frac{\partial Q_n(r)}{\partial r_n} > 0$, $\forall n \in [N(r)]$.

And now consider the following model:

Definition 2. A quantal response Q admits a control costs representation iff there is a convex $c : [0, 1] \to \mathbb{R} \cup \{\infty\}$, strictly convex and $\mathscr{C}^2$ over $(0, 1)$, satisfying $\lim_{p \to 0} c'(p) = -\infty$, such that,

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for any $r \in \mathscr{R}$, $Q(r) = \arg\max_{\sigma \in \Delta([N(r)])} \sum_{\ell \in [N(r)]} \sigma(\ell) r_\ell - c(\sigma(\ell))$.

If Q admits a control costs representation, then it is said to be a control costs quantal response (CCQR).

Question 2.(a) Prove the following statement: If Q is a control costs quantal response, then Q is regular.

Question 2.(b) Suppose Q is a control costs qual response. Prove the following claims:

(i) Q satisfies translation invariance: $Q(r) = Q(r + k)$, for any constant vector $k \in \mathbb{R}^{N(r)}$;

(ii) Q satisfies stochastic monotonicity: $\forall r, r' \in \mathscr{R}$, if $N(r) \leq N(r')$ and $r_n = r'_n$ $\forall n \in [N(r)]$, then $Q_n(r') \leq Q_n(r)$ $\forall n \in [N(r)]$; and

(iii) Q satisfies strong stochastic transitivity: $\forall x, y, z \in \mathbb{R}$, if $Q_1((x,y)), Q_1((y,z)) \geq 1/2$, then $Q_1(x,z) \geq \max\{Q_1((x,y)), Q_1((y,z))\}$.

Question 2.(c) A buyer with control costs values an item at v and is choosing between buying at price p from the seller, and not buying a product, getting a payoff of 0. In short, the buyer's problem is summarised by $r = (v - p, 0)$.

Suppose the seller knows the buyer's value v and the buyer's control costs c. Characterise the price p^* that maximises the seller's expected revenue (the price times the probability of sale at that price).

Question 2.(d) If v increases, what happens to the seller's maximised expected revenue?